

Table 2.3
Auxiliary functions after Holstein and Bohlen [7]

Λ	K	$f_1(K)$	$f_2(K)$
12	0.0948	2.250	0.356
10	0.0919	2.260	0.351
8	0.0831	2.289	0.340
7.6	0.0807	2.297	0.337
7.2	0.0781	2.305	0.333
7.0	0.0767	2.309	0.331
6.6	0.0737	2.318	0.328
6.2	0.0706	2.328	0.324
5.0	0.0599	2.361	0.310
3.0	0.0385	2.427	0.283
1.0	0.0135	2.508	0.252
0	0	2.554	0.235
-1	-0.0140	2.604	0.217
-3	-0.0429	2.716	0.179
-5	-0.0720	2.847	0.140
-7	-0.0999	2.999	0.100
-9	-0.1254	3.176	0.059
-11	-0.1474	3.383	0.019
-12	-0.1567	3.500	0

Thwaites [9] modified and improved the method due to Holstein and Bohlen [7]. He rewrote the momentum integral equation in terms of a parameter, λ , defined as

$$\lambda = \frac{(\delta^{**})^2 U'}{\nu} = \left(\frac{\delta^{**}}{\delta} \right)^2 \Lambda$$

The method due to Thwaites is not discussed in this text. The advanced readers may like to refer to the original work of Thwaites [9].

2.10 EFFECT OF PRESSURE GRADIENT ON EXTERNAL FLOWS

For the case of a boundary layer on a flat plate, the pressure gradient of the external stream is zero. Let us consider a body with curved surface (Fig. 2.15). Upstream of the highest point the streamlines of the outer flow converge, resulting in an increase of the free stream velocity $U(x)$ and a consequent fall of pressure with x . Downstream of the highest point the streamlines diverge, resulting in a decrease of $U(x)$ and a rise in pressure. In this section we shall investigate the effect of such a pressure gradient on the shape of the boundary layer profile $u(x, y)$. The boundary layer equation is

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2}$$

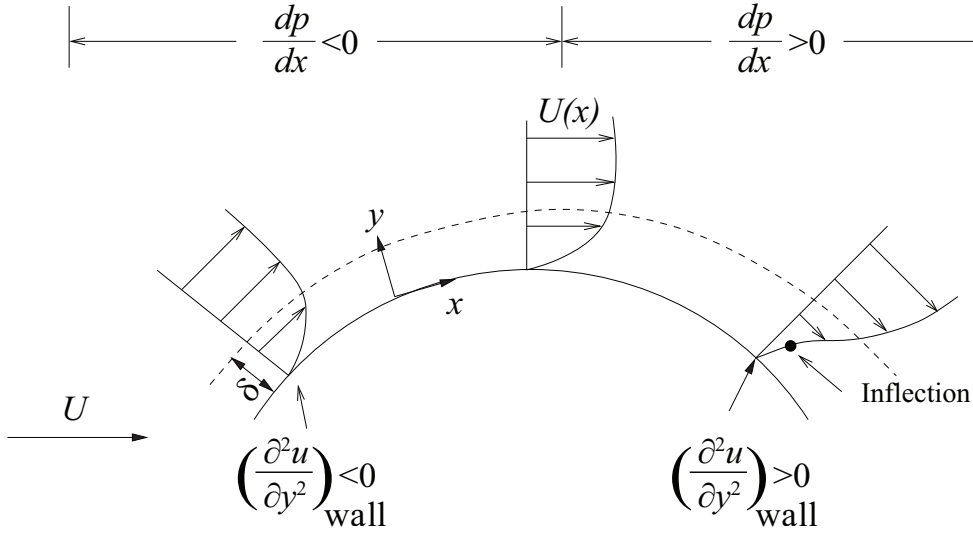


Figure 2.15 Velocity profile associated with separation in a cross flow over a circular cylinder

where the pressure gradient is found from the external velocity field as $dp/dx = -\rho U(dU/dx)$, with x taken along the surface of the body. At the wall, the boundary layer equation becomes

$$\mu \left(\frac{\partial^2 u}{\partial y^2} \right)_w = \frac{\partial p}{\partial x} \quad (2.161)$$

In an accelerating stream $dp/dx < 0$ (see Fig. 2.15),

$$\left(\frac{\partial^2 u}{\partial y^2} \right)_{\text{wall}} < 0 \quad (\text{accelerating}) \quad (2.162)$$

As the velocity profile has to merge smoothly with the external flow, the slope $\partial u / \partial y$ slightly below the edge of the boundary layer decreases with y from a positive value to zero. Therefore, $\partial^2 u / \partial y^2$ slightly below the boundary layer edge is negative. Eq. (2.162) then shows that $\partial^2 u / \partial y^2$ has the same sign at both the wall and the boundary layer edge, and presumably throughout the boundary layer (Fig. 2.16). In contrast, for a decelerating external stream, the curvature of the velocity profile at the wall is

$$\left(\frac{\partial^2 u}{\partial y^2} \right)_{\text{wall}} > 0 \quad (\text{decelerating}) \quad (2.163)$$

so that the curvature changes sign somewhere within the boundary layer. In other words, the boundary layer profile in a decelerating flow ($dp/dx > 0$) has a point of inflection where $\partial^2 u / \partial y^2 = 0$ (Fig. 2.17).

The shape of the velocity profiles in the figures suggests that *an adverse pressure gradient tends to increase the thickness of the boundary layer*. This can also be seen from the continuity equation.

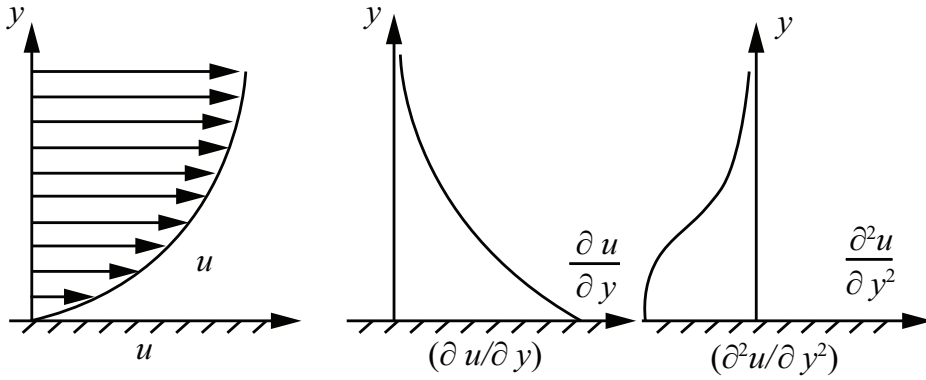


Figure 2.16 Profiles for $dp/dx < 0$ (favorable pressure gradient)

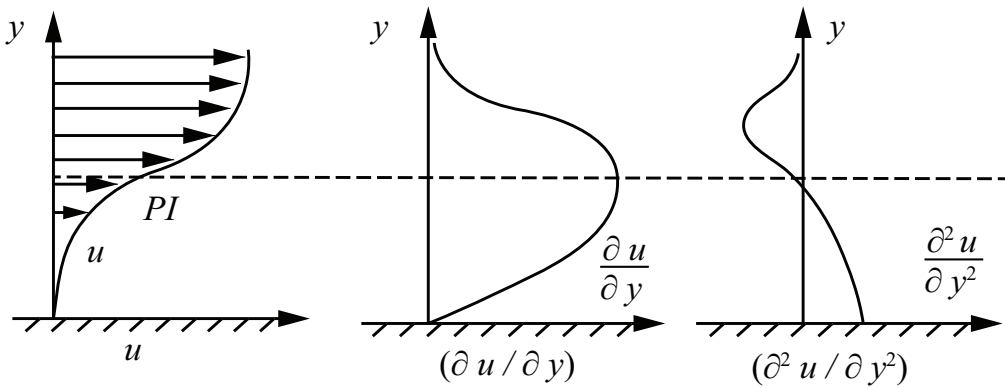


Figure 2.17 Profiles for $dp/dx > 0$ (adverse pressure gradient)

$$v(y) = - \int_0^y \frac{\partial u}{\partial x} dy \quad (2.164)$$

Compared to a flat plate, a decelerating external stream causes a larger $-\partial u/\partial x$ within the boundary layer because the deceleration of the outer flow adds to the viscous deceleration within the boundary layer. From the above equation we observe that the v -field, directed away from the surface, is larger for a decelerating flow. The boundary layer therefore thickens not only by viscous diffusion but also by advection away from the surface, resulting in a rapid increase in the boundary layer thickness with x . If pressure falls along the direction of flow, $dp/dx < 0$ and we say that the pressure gradient is favorable. If, on the other hand, the pressure rises along the directions of flow, $dp/dx > 0$ and we say that the pressure gradient is adverse. The rapid growth of the boundary layer thickness in a decelerating stream,

and the associated large v -field, causes the important phenomena of separation, in which the external stream ceases to flow nearly parallel to the boundary surface.

2.11 DESCRIPTION OF FLOW PAST CIRCULAR CYLINDER

Let us start with a consideration of the creeping flow around a circular cylinder, characterized by $Re < 1$. (Here we shall define $Re = U_\infty d/\nu$, based on the upstream velocity and the cylinder diameter.) Vorticity is generated close to the surface because of the no-slip boundary condition. In the Stokes approximation this vorticity is simply diffused, not advected, which results in a fore and aft symmetry.

As Re is increased beyond 1, the vorticity is increasingly confined behind the cylinder because of advection. For $Re > 4$ two small attached or standing eddies appear behind the cylinder. The wake is completely laminar and the vortices act like rollers over which the main stream flows (Fig. 2.18). The eddies get larger as Re is increased. A very interesting sequence of events begins to develop when

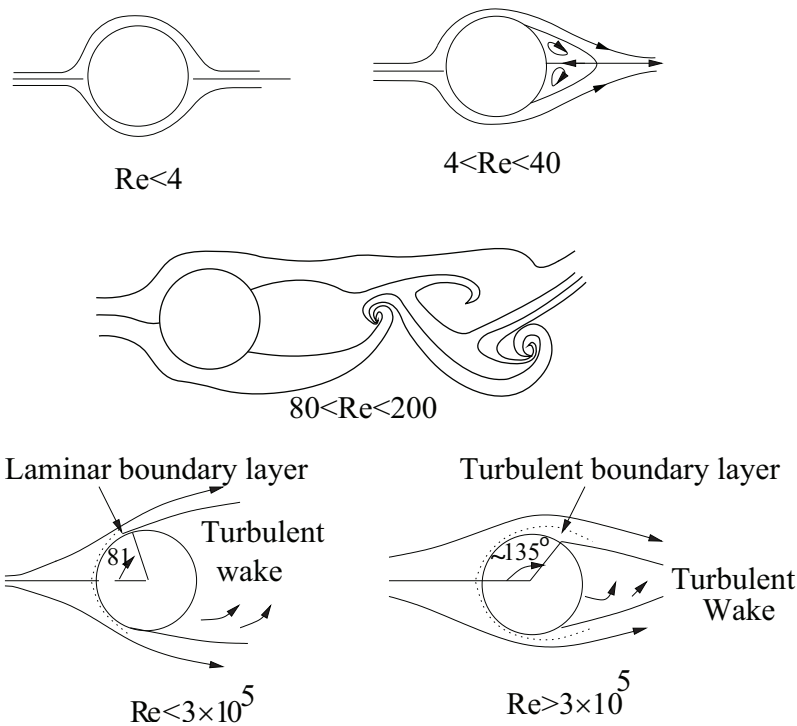


Figure 2.18 Influence of Reynolds number on the wake of a circular cylinder

the Reynolds number is increased beyond 40, at which point the wake behind the cylinder becomes unstable. The eddies start to oscillate in time and asymmetry is brought about in the wake structure. The wake develops a slow oscillation in which the velocity is periodic in time, with the amplitude of the oscillation increasing downstream. The oscillating wake starts shedding two staggered rows of vortices with opposite sense of rotation, in the stream. Karman investigated the phenomena

and concluded that a nonstaggered row of vortices is unstable, and a staggered row is stable only if the ratio of lateral distance between the vortices to their longitudinal distance is 0.28. The staggered row of vortices behind a blunt body is called a von Karman vortex street. The structure is amazingly stable and it is possible to see a large number of vortices downstream of the body. The vortices move downstream at a speed smaller than the upstream velocity U . At this stage periodicity is induced in the entire flow field due to the vortex shedding phenomena. During the vortex shedding process, when an eddy on one side is shed, another forms on the other side, resulting in an unsteady flow near the cylinder. As vortices of opposite circulations are shed off alternately from the two sides, the circulation around the cylinder changes sign, resulting in an oscillating lift or lateral force. If the frequency of vortex shedding is close to the natural frequency of some mode of vibration of the cylinder body, then an appreciable lateral vibration takes place. You may recall the collapse of the Tacoma Narrows Bridge in Washington State in the U.S. A torsional mode of oscillation in the structure synchronized with the vortex shedding frequency to build up the oscillations to destructive magnitude. Engineering structures such as chimney stacks, suspension bridges and oil drilling platforms are designed to break up a coherent shedding of vortices from the cylindrical structures. This is done by using helical strakes on the cylinder surface, which break up the spanwise coherence of shedding of the structure.

The passage of regular vortices causes velocity in the wake to have a dominant periodicity. The frequency f is expressed as a nondimensional parameter known as the Strouhal number, named after V. Strouhal, a German engineer. The Strouhal number is defined as

$$S \equiv \frac{fd}{U}$$

For a circular cylinder the value of S remains close to 0.21 for a large range of Reynolds numbers. For small values of cylinder diameter and moderate values of U , the resulting frequencies of the vortex shedding and oscillating lift lie in the acoustic range. For example, at $U = 10$ m/s and a wire diameter of 2 mm, the frequency corresponding to a Strouhal number of 0.21 is $f = 1050$ cycles per second and the corresponding Reynolds number is 1200.

Strouhal made his first measurements in 1878. Roshko, in 1954, measured the frequencies using a hot-wire probe. For low Reynolds number laminar regions Roshko consolidated his results to an equation of the form

$$S = 0.212(1 - 21.2/Re) \quad (2.165)$$

At about Reynolds number of 500, multiple frequencies start to appear and the wake tends to become turbulent.

2.12 EXPERIMENTAL RESULTS FOR CIRCULAR CYLINDER FLOW

In this section, we analyze the heat transfer results for the case of flow over a heated circular cylinder (Fig. 2.19). In order to understand heat transfer at a moderate

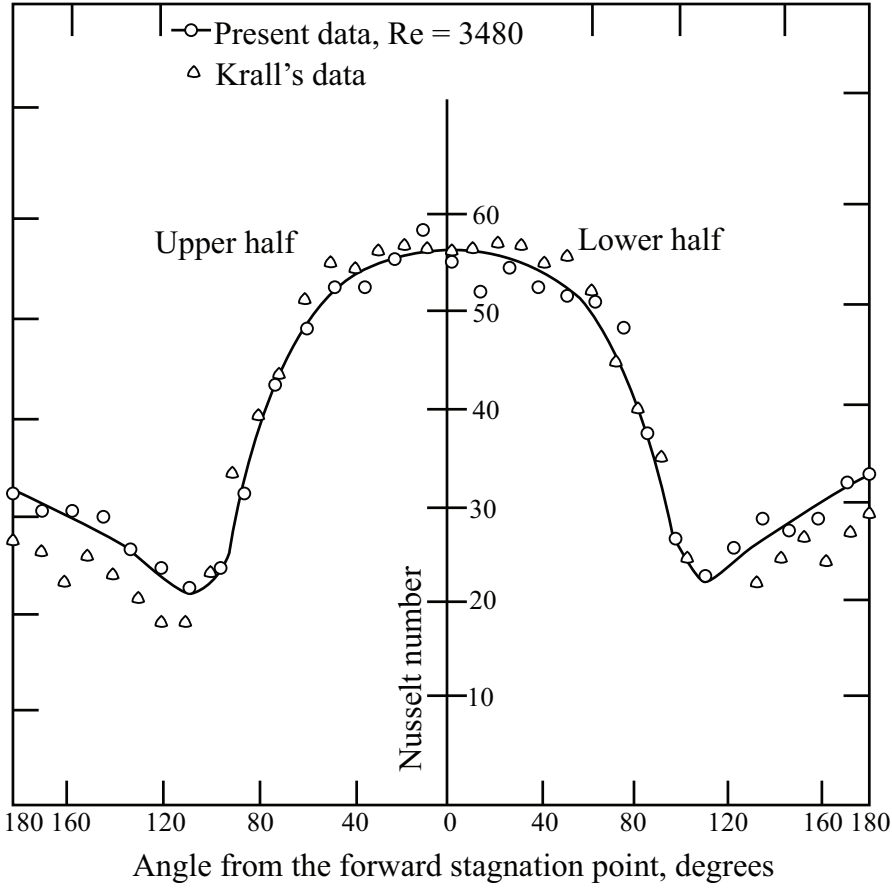


Figure 2.19 Typical distribution of local Nusselt number in forced convection, $Re = 3480$ (courtesy of Sarma and Sukhatme [10])

Reynolds number ($Re = 3480$), we take a recourse to the investigation of Sarma and Sukhatme [10]. Starting with a high value at the stagnation point, Nu_θ decreases with increasing θ as a result of laminar boundary layer development. The minimum is reached at the point of separation. The point of separation is symmetrical about the forward stagnation point and occurs at about 110° . After the separation point, Nu_θ increases with θ because of the mixing associated with the vortex formation in the wake. It may be mentioned that the values of Nu_θ are the time-averaged quantities. Sarma and Sukhatme [10] also found the correlation between the Nusselt number at the forward stagnation point and Reynolds number as

$$Nu_{fsp} = 0.91(Re)^{0.5} \quad (2.166)$$

Since working fluid was air only, the variation of Prandtl number was not considered. Next we take up the heat transfer for higher Reynolds numbers [11]. Consider the results for $Re_D < 10^5$ (Figure 2.20). Following the earlier trend, the Nusselt

number is high at the forward stagnation point. The Nu_θ reduces with the growth of boundary layer and reaches a minimum at $\theta = 80^\circ$. At this point, separation occurs, and Nu_θ increases with θ because of the mixing due to eddies in the wake. In contrast, for $Re_D > 10^5$ the variation of Nu_θ with θ is characterized by two minima. The decline in Nu_θ from the value at the stagnation point is again due to laminar boundary layer development, but the sharp increase that occurs between 80° and 100° is owing to the boundary layer transition to turbulence. With further development of the turbulent boundary layer, Nu_θ again begins to decline. Eventually turbulent boundary layer separation occurs at $\theta \approx 140^\circ$ and then Nu_θ increases as a result of considerable mixing associated with the wake region.

Overall average condition is:

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = C Re_D^m Pr^{1/3} \quad (\text{due to Hilpert [12]}) \quad (2.167)$$

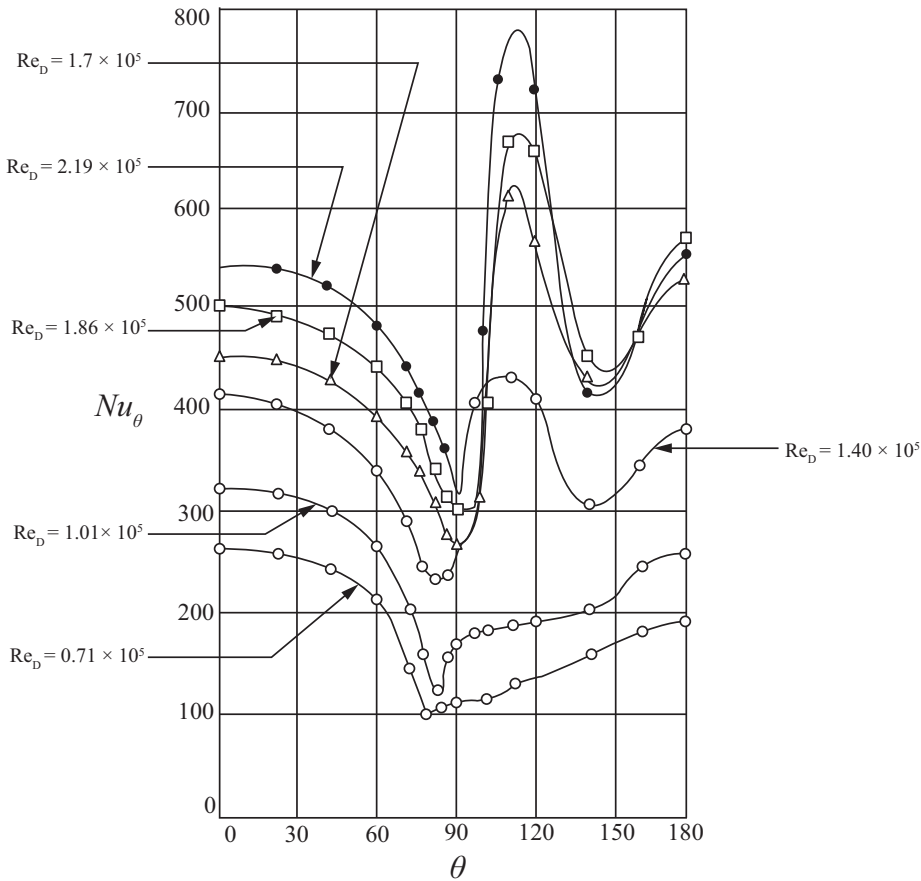


Figure 2.20 Local Nusselt number distribution for flow over a circular cylinder for different Reynolds numbers (courtesy of Giedt [11])

A more recent general correlation is:

$$\overline{Nu}_D = C Re_D^m Pr^n \left(\frac{Pr_\infty}{Pr_w} \right)^{1/4} \quad (\text{by Zhukauskas [13]}) \quad (2.168)$$

The values of C and m are given in Table 2.4.

Table 2.4 [12,14]

Re_D	C	m
4 - 40	0.911	0.385
40 - 4000	0.683	0.466
4000 - 40000	0.193	0.618
40000 - 400000	0.027	0.805

- All properties are evaluated at T_∞ , except Pr_w , which is evaluated at T_w . For $Pr \leq 10$, $n = 0.37$ and for $Pr > 10$, $n = 0.36$.

2.13 OTHER IMPORTANT CORRELATIONS

For small Prandtl number fluids (liquid metals) the local heat transfer coefficient on a flat plate is given by

$$Nu_x = 0.565 Pe_x^{1/2}, \quad \text{for} \quad Pe_x \leq 0.01 \quad (2.169)$$

From experiments (see Schlichting [3]) the local friction coefficients for turbulent flow over a flat plate are given by

$$C_{f\ x} = 0.0592 Re_x^{-1/5} \quad 5 \times 10^5 < Re_x < 10^7 \quad (2.170)$$

To a reasonable approximation, the velocity boundary layer thickness for turbulent flow over a flat plate may be expressed as

$$\frac{\delta}{x} = \frac{0.37}{(Re_x)^{-1/5}} \quad (2.171)$$

(Observe that δ varies as $x^{4/5}$ in contrast to $x^{1/2}$ for laminar flow.) The local Nusselt number for turbulent flow over a flat plate is:

$$Nu_x = 0.0296 Re_x^{4/5} Pr^{1/3} \quad (2.172)$$

In mixed boundary layer situations, the average heat transfer coefficient for the

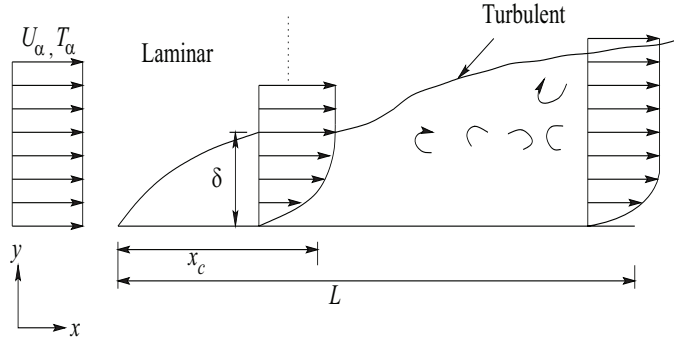


Figure 2.21 Transitional flow over a flat plate

flow over a flat plate (Fig. 2.21) is given by

$$\bar{h}_L = \frac{1}{L} \left(\int_0^{x_c} h_{lam} dx + \int_{x_c}^L h_{turb} dx \right) \quad (2.173)$$

or

$$\bar{h}_L = \frac{k}{L} \left[0.332 \left(\frac{U_\infty}{\nu} \right)^{1/2} \int_0^{x_c} \frac{dx}{x^{1/2}} + 0.0296 \left(\frac{U_\infty}{\nu} \right)^{4/5} \int_{x_c}^L \frac{dx}{x^{1/5}} \right] Pr^{1/3}$$

or

$$\overline{Nu}_L = \left[0.664 Re_{x,c}^{1/2} + 0.037 \left(Re_L^{4/5} - Re_{x,c}^{4/5} \right) \right] Pr^{1/3}$$

or

$$\overline{Nu}_L = \left(0.037 Re_L^{4/5} - A \right) Pr^{1/3} \quad (2.174)$$

If the typical, transition Reynolds number is $Re_{x,c} = 5 \times 10^5$

$$\overline{Nu}_L = \left(0.037 Re_L^{4/5} - 871 \right) Pr^{1/3} \quad (2.175)$$

Similarly,

$$\overline{C}_{f,L} = \frac{0.074}{Re_L^{1/5}} - \frac{A}{Re_L} \quad (2.176)$$

where $A = 1742$.

If $L \gg x_c$ ($Re_L \gg Re_{x,c}$)

$$\overline{Nu}_L = 0.037 Re_L^{4/5} Pr^{1/3} \quad \text{and} \quad \overline{C}_{f,L} = 0.074 Re_L^{-1/5}$$

We know that the fluid properties vary with temperature across the boundary layer and that this variation can certainly influence heat transfer. This influence may be

handled by one of the following two ways. In one method all the properties evaluated at a mean boundary layer temperature T_f , termed the film temperature.

$$T_f = \frac{T_w + T_\infty}{2}$$

In the alternate method, evaluate all the properties at T_∞ and multiply the right-hand side of the expression for Nusselt number by an additional parameter. This parameter is commonly of the form $(Pr_\infty/Pr_w)^r$ or $(\mu_\infty/\mu_w)^r$ where w designates *properties at surface temperature*.

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